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"Isometries of Wasserstein spaces and Lipschitz-free spaces"

Due to its nice geometric properties and an astonishing number of applications, probably one of the most intensively studied metrics nowadays is the p-Wasserstein metric. Given a complete and separable metric space M and a positive real number p, one defines the p-Wasserstein space $\mathcal{W}_p(M)$ as the set of all Borel probability measures with finite p-th moment, endowed with a metric which is calculated using optimal transport plans.

In this talk, I will focus on isometries of $\mathcal{W}_p(M)$ spaces, mainly in the case when $p = 1$ – the case which is closely connected to the theory of Lipschitz-free spaces.

It is known that if F is an isometry of M, then its push-forward $F_\#$ is an isometry of $\mathcal{W}_p(M)$. In other words, the isometry group of M embeds into the isometry group of $\mathcal{W}_p(M)$. A natural question arises : is this embedding surjective ? In other words : are the isometry groups $\mathrm{Isom}(M)$ and $\mathrm{Isom}(\mathcal{W}_p(M))$ isomorphic ? We know several concrete examples where the answer is yes (see e.g. [1,4,6]), but the answer, in general, is no (see e.g. [2,3,5]). This talk aims to discuss some special cases and to shed some light on similarities and differences between isometries of Wasserstein spaces and Lipschitz-free spaces.

REFERENCES

[1] J. Bertrand and B. Kloeckner, A geometric study of Wasserstein spaces : isometric rigidity in negative curvature, *Int. Math. Res. Not.* 2016(5) :1368–1386, 2016.

[2] Gy.P. Gehér, T. Titkos, and D. Virosztek, Isometric study of Wasserstein spaces–the real line, *Trans. Amer. Math. Soc.* 373 :5855–5883, August 2020.

[3] Gy.P. Gehér, T. Titkos, and D. Virosztek, The isometry group of Wasserstein spaces : the Hilbertian case, *J. London Math. Soc.* 2022, doi :10.1112/jlms.12676.

[4] G. Kiss and T. Titkos, Isometric rigidity of Wasserstein spaces : the graph metric case, *Proc. Amer. Math. Soc.*, 150(9) :4083–4097, 2022.

[5] B. Kloeckner. A geometric study of Wasserstein spaces : Euclidean spaces, *Ann. Sc. Norm. Super. Pisa Cl. Sci.* (5), 9(2) :297–323, 2010.

[6] J. Santos-Rodríguez. On isometries of compact Lp-Wasserstein spaces, *Adv. Math.* 409 :Paper No. 108632, 21, 2022.